

the case  $R_1 = 0$ , it follows from an analytic solution of the electrotechnical loop equations that the main fraction of the energy stored in the transformer inductances remains in the primary loop. Only the energy stored in the load inductance is liberated in the second loop resistance for  $t > t_m$ , and there is no negative phase. In the more general case under consideration  $R_1 \neq 0$ , a part of the primary contour energy is indeed transmitted to the load.

Computations showed that the dependence of the energy liberated in the load on its resistance has a maximum which shifts toward greater values of  $R_L$  as the inductance of the secondary winding of the matching transformer increases. The greatest value of the energy (240 kJ) was obtained in test 5 in which the resistance is close to the optimal computed value  $R_{opt} \approx 10^{-1} \Omega$ . The possibility of a further increase in the active energy during diminution of the load inductance and by a proportional increase in  $L_2$  and  $R_L$  for a given  $L_L$  follows from the computations.

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#### HEAT AND MASS TRANSFER OF AN AEROSOL DURING MIXING WITH THE FLUX OF A NONEQUILIBRIUM VIBRATIONALLY EXCITED MEDIUM

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The use of mixing of vibrationally excited nitrogen and carbon dioxide aerosol fluxes in order to improve the energetic characteristics and increase the homogeneity and volume of an active inversion medium was proposed in [1, 2]. It was assumed that the insertion of a solid phase  $CO_2$  in a supersonic nitrogen stream in the initial stage might be performed by mechanical means without using the mechanisms of laminar and turbulent diffusion, whereupon mixing of the subsonic and supersonic fluxes is usually realized. Then because of the latent heat of sublimation of the solid phase, an additional reduction in the translational-rotational temperature of nitrogen and of the symmetric and deformational vibrational degrees of freedom of the  $CO_2$  molecules can be achieved for a given degree of expansion of the main flow. The stagnation temperature and the nitrogen pressure in the forechamber can also evidently be raised by this method, and therefore, the population inversion is increased both because of the rise in the upper level vibrational temperature and because of the reduction in the lower level temperature. In order to realize these possibilities, the processes of aerosol interaction with a vibrationally excited gas must be analyzed thoroughly.

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The mixing problem is considered in this paper on the basis of using the conservation equations. Necessary conditions are determined for realizing the "instantaneous" mixing of aerosol particles with the flux of a vibrationally excited medium. Finally, estimating formulas are given to determine the particle lifetime, and their depth of penetration for the continuous-medium, transition, and free-molecule flow mixing modes.

1. Let us analyze the mixing process on the basis of the mass, momentum, and energy conservation equations for the flow in a constant-cross-section channel. We write them as follows [1] by assuming for definiteness that the solid phase carrier is carbon dioxide which mixes with the nitrogen:

$$\begin{aligned} \frac{m_p v_p^2}{2} + \frac{m_g v_g^2}{2} + c_p^{(g)} m_g T_g + c_p^{(p)} m_p T_p &= \frac{(m_p + m_g) v^2}{2} + c_p^{(g)} \left( \frac{m_p}{\mu_p} \mu_g + m_g \right) T + q m_p, \\ \frac{m_p^*}{\mu_p} \frac{RT_p}{v_p} + m_p v_{pt} + \frac{m_g}{\mu_g} \frac{RT_g}{v_g} + m_g v_g &= \left( \frac{m_p}{M_p} M_g + m_g \right) \frac{RT}{\mu_g v} + (m_p + m_g) v, \end{aligned} \quad (1.1)$$

where  $c_p^{(g)} = (\gamma/\gamma - 1)(R/\mu_g)$ ;  $\gamma = c_p^{(g)}/c_v^{(g)}$ ;  $R = kN_A$  is the universal gas constant,  $\mu_g = M_g N_A$  is the weight of one mole of gas,  $k$  is the Boltzmann constant,  $N_A$  is the Avogadro number,  $m_p^* = N_p^*(M_p v_{ps})$ ;  $N_p^*$  is the initial vapor concentration in the section  $S_p$  prior to mixing, and the subscripts  $g$  and  $p$  refer to the gas (nitrogen), and the particles (carbon dioxide), respectively.

Let us go over to dimensionless variables

$$\begin{aligned} w_{pt} &= v_{pt}/v_g, \quad w_{pn} = v_{pn}/v_g, \\ w &= v/v_g, \quad \Theta_p = T_p/T_g, \quad t = T/T_g, \end{aligned}$$

where  $t$ ,  $n$  denote the tangential and normal components of the velocity, and let us introduce the dimensionless parameters

$$\begin{aligned} \varepsilon &= \frac{m_p}{m_g} = \frac{N_p^0 M_p v_{ps}}{N_g^0 M_g v_{gs}}, \quad Q = \frac{q}{c_p^{(g)} T_g}, \\ \bar{m} &= \frac{M_g}{M_p}, \quad \bar{c} = \frac{c_p^{(p)}}{c_p^{(g)}}, \quad M = \frac{v_g}{a_g}, \quad a_g = \sqrt{\gamma \frac{R}{\mu_g} T_g}. \end{aligned}$$

Neglecting the vapor density prior to mixing  $N_p^*$  in (1.1) henceforth and assuming that the vertical component of the momentum is quenched by the wall reaction, we obtain the following solutions for the parameters after mixing:

$$w = \frac{\gamma \left( 1 + \frac{1}{\gamma M^2} + \varepsilon w_{pt} \right)}{(\gamma + 1)(1 + \varepsilon)} \pm (\gamma + 1)^{-1} (1 + \varepsilon)^{-1} (SQRT), \quad p = (\varepsilon \bar{m} + 1) \frac{t}{w}; \quad (1.2)$$

$$t = \frac{\gamma(\gamma - 1)M^2}{(\gamma + 1)(1 + \varepsilon m)} \left[ \varepsilon (w_{pt}^2 + w_{pn}^2) + 1 + \frac{2(1 + \varepsilon \bar{c} \Theta_p)}{(\gamma - 1)M^2} - \frac{2\varepsilon Q}{(\gamma - 1)M^2} - \frac{\gamma \left( \frac{1}{\gamma M^2} + 1 + \varepsilon w_{pt} \right)^2}{(\gamma + 1)(1 + \varepsilon)} - \frac{\left( \frac{1}{\gamma M^2} + 1 + \varepsilon w_{pt} \right) SQRT}{(\gamma + 1)(1 + \varepsilon)} \right],$$

where

$$SQRT = \sqrt{\left( \frac{1}{M^2} - 1 \right)^2 - \varepsilon \left\{ (\gamma^2 - 1) [(w_{pt} - 1)^2 + (1 + \varepsilon) w_{pn}^2] - 2w_{pt} \left( 1 + \frac{\gamma}{M^2} \right) - \varepsilon w_{pt}^2 + \frac{2(\gamma - 1)}{M^2} A \right\}};$$

$$A = [1 + \bar{c} \Theta_p - Q + \varepsilon (\bar{c} \Theta_p - Q)].$$

We select the branch of the solution corresponding to the plus sign in (1.2), i.e., as  $\varepsilon \rightarrow 0$  we take  $w \rightarrow 1$ .<sup>†</sup> As  $\gamma \rightarrow 1$  and  $M \rightarrow \infty$  these solutions agree with those obtained in [1] and correspond to neglecting the pressure in comparing the dynamic head in the momentum equation. Using (1.2) and (1.3), we analyze the different interaction modes between the two fluxes and determine the mixture flow parameters being formed because of the mixing.

<sup>†</sup>In principle, if a strong shock occurs during mixing then the subsonic branch of the solution holds. Realization of this case is of no interest for the problem under consideration here.

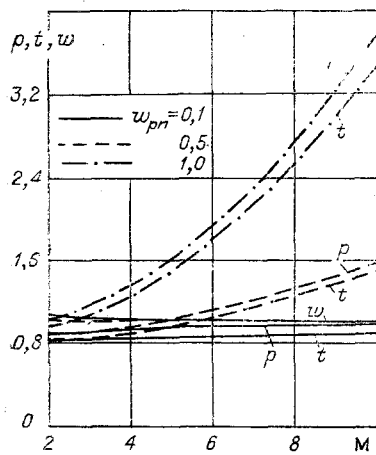


Fig. 1

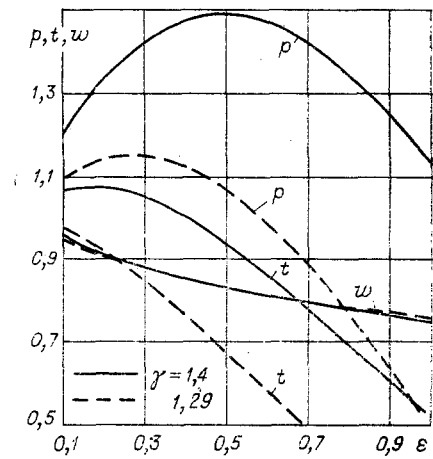


Fig. 2

The quantities  $p$ ,  $t$ ,  $w$  are shown in Fig. 1 for a quasicomixing  $w_{pt} \sim 1$  and  $\gamma = 1.4$ ,  $\epsilon = 0.1$ ,  $\Theta_p = 1$ ,  $Q = 2$ ,  $\bar{c} = 1$ ,  $\bar{m} = 0.635$ . It is seen that the values of  $p$  and  $t$  increase because of the mixing, and all the more, the greater the normal component of the aerosol particle velocity  $w_{pn}$ . An analogous result holds if  $w_{pt}$  varies for a fixed  $w_{pn}$ . This indicates that it is hardly possible to organize a completely homogeneous flow because of the interaction since a pressure rise should be related to the origination of a compression wave. The velocity is a more conservative function and does not actually vary as the number  $M$  increases in the stream. As the quantity of delivered mass  $\epsilon$  increases, as a rule considerable pressure and temperature deviations from the initial values  $p = 1$ ,  $t = 1$  are observed behind the mixing zone, i.e., there is a strong inhomogeneity in the mixing process. However, minimal changes in certain quantities in the mixing process can be achieved for a special selection of the parameters (for instance, for  $w_{pt} = 0.5$ ,  $w_{pn} = 0.1$ , Fig. 2). Thus, it is seen in Fig. 2 that for  $\epsilon = 0.4$  the pressure equals the initial value and  $t < 1$ . However, such a mode is spoiled for comparatively small changes in the parameters  $w_{pt}$ ,  $w_{pn}$ , etc. Hence, to assure given mixing conditions the gasdynamic flows should be tuned to "resonance." An increase in the quantity of delivered mass to the value  $\epsilon \approx 0.4$  is of interest in the following respects. The active particle transit velocity through a resonator in a hypersonic laser is higher compared to traditional GDL types, therefore, other conditions being equal, the coefficient of  $\text{CO}_2$  molecule utilization drops. Hence, an increase in the fraction of  $\text{CO}_2$  by about 10% in comparison to the quantity ordinarily used can turn out to be expedient. As is shown in [3], this is also essential because the particle-size distribution functions have a "heavy" tail. The main part of the aerosol particles responsible for formation of the active medium is hence a small part of the total mass of particles delivered to the gas. Another additional advantage of the increase in  $\epsilon$  is the reduction in the temperature  $t$  which is close to the temperature of the lower lasing level. The main disadvantage here is the substantial increase in the quantity of working body needed to accelerate the particles. Let us note that after mixing the mass of aerosol vapors equals the initial mass of the solid phase in the formulation based on using the conservation equations in the form (1.1). However, complete evaporation is actually realized not for all values of the parameter  $\epsilon > 0$ . It ceases for a certain  $\epsilon \geq \epsilon_{kp}$ , when the phase equilibrium conditions, i.e., the equality of the saturated vapor temperature and concentration on the particle surface and in the gas phase, are established prior to total particle evaporation. The quantity  $\epsilon_{kp}$ , which is a function of the initial gas temperature  $T_g$  can be found by using tables [4]. For the conditions presented in Fig. 2 and  $T_g = 300^\circ\text{K}$ , the quantity is  $\epsilon_{kp} \approx 0.7$ . However even in those cases when  $\epsilon < \epsilon_{kp}$  and the solid phase evaporates completely, the resultant mixing parameters cannot satisfy the demands imposed on the active medium (for instance, at the entrance to the resonator). It is necessary that evaporation (total or partial) and mixing of the  $\text{CO}_2$  vapor being formed with the nitrogen occur during a time less than the vibrational relaxation time of the mixture  $\text{CO}_2 + \text{N}_2$ . The particle size, the depth of their penetration, and their lifetime should hence also be selected in an appropriate manner. An analysis based on the use of the conservation equations cannot yield an answer to these questions, and it is hence expedient to investigate particle motion and their evolution in the vibrationally excited medium in greater detail.

2. Let us isolate the characteristic features of this problem which have yet been studied only slightly and not received proper illumination in the extensive literature devoted

to investigations of multiphase flows [5]. Among these features is the problem of the influence of the vibrational nonequilibrium of the supersonic flow on the rate of evaporation of the particles inserted therein, and also the determination of the lifetime and depth of penetration of particles of different size in a flow with the arbitrary Knudsen number  $\epsilon_g$ . In this connection, we note that such a problem was examined earlier in [6, 7] only for an equilibrium gas without internal degrees of freedom and in a bounded range of numbers  $\epsilon_g$ .

In the case under consideration, the equations for the temperature change  $\Theta_p$ , the radius  $R$ , and the particle velocity  $V$  written in dimensionless form are:

$$\frac{d\Theta_p}{d\tau} = \frac{3}{2} \frac{\gamma_p}{R^2} \left[ \text{Nu}(1 - \Theta_p) + \text{Nu}_v(\Theta_{vg} - \Theta_p) \frac{\kappa_v}{\kappa_g} - \text{Le} Q \text{Sh}(\alpha^* - \alpha) \right]; \quad (2.1)$$

$$\frac{dR}{d\tau} = -\frac{\text{Le}}{2R} \text{Sh}(\alpha^* - \alpha); \quad (2.2)$$

$$\frac{dV}{d\tau} = -\frac{3 \text{Pe}}{16R} c_D V^2, \quad (2.3)$$

where  $R = r_p/r_{p0}$ ,  $V = |v_p - v_g|/|v_p - v_g|_{\tau=0}$ ,  $r_{p0}$ ,  $|v_p - v_g|_{\tau=0}$  are the radius and particle velocity at the initial time  $\tau = 0$ ;  $\Theta_{vg} = T_v/T_g$  is the dimensionless temperature of the vibrational degrees of freedom of the gas molecules,  $\gamma_p = c_p(g)/c_p(p)$  is the ratio of the specific heats of the gas and solid phases,  $\text{Pe}$  is the Peclet number,  $\text{Nu}$ ,  $\text{Nu}_v$ ,  $\text{Sh}$ ,  $c_D$  are the dimensionless heat transfer, mass transfer, and drag coefficients,  $\tau$  is the dimensionless time of particle motion on the trajectories ( $\tau = (\rho_g/\rho_p)F_0$ ,  $F_0 = \chi_g t/r_{p0}^2$  is the Fourier number,  $\chi_g$  is the coefficient of thermal diffusivity in the gas phase,  $\rho_g/\rho_p$  is the ratio of the gas and solid phase densities),  $\kappa_v$ ,  $\kappa_g$  are the vibrational and translational-rotational coefficients of heat conduction in the gas,  $\alpha^*$  is the saturated vapor concentration on the particle surface,  $\alpha$  is the vapor concentration in the gas phase, and henceforth the parameters  $\alpha$  and  $T_g$  are assumed known. The heat flux of the vibrational degrees of freedom of the gas molecules, proportional to the number  $\text{Nu}_v$  introduced by analogy with the usual Nusselt number is extracted as a separate component in the energy equations (2.1). This is because the heat flux is determined by the sum of the components  $\kappa_g \partial T_g / \partial r + \kappa_v \partial T_v / \partial r$ , referred to the different degrees of freedom [8], in a nonequilibrium vibrationally excited gas when  $T_v \neq T_g$ , particularly  $T_v \gg T_g$ .

For arbitrary values of the dimensionless parameters in (2.1)-(2.3), their solution is a sufficiently complex problem. Hence, we first investigate the particular class of solutions (2.1)-(2.3) corresponding to the gas at rest, i.e.,  $\text{Re} \rightarrow 0$ . To determine the dependences of the dimensionless coefficients  $\text{Nu}$ ,  $\text{Nu}_v$ , etc. on the Knudsen number  $\epsilon_g$  according to the particle size and the other parameters, we use the approximate method of the boundary sphere [9]. We then obtain

$$\text{Nu}(R, \epsilon_g, \alpha_T) = \frac{2\alpha_T(R + \epsilon_g)R}{\alpha_T R^2 + \epsilon_{Tg}(R + \epsilon_g)}, \quad (2.4)$$

$$\text{Nu}_v(R, \epsilon_g, \alpha_v) = \frac{2\alpha_v(R + \epsilon_g)R}{\alpha_v R^2 + \epsilon_{vg}(R + \epsilon_g)}, \quad \text{Sh}(R, \epsilon_g, \beta) = \frac{2\beta(R + \epsilon_g)R}{\beta R^2 + \epsilon_{Dg}(R + \epsilon_g)},$$

where

$$\epsilon_g = \frac{2v_g}{3v_T r_{p0}}, \quad v_T = \sqrt{\frac{2kT_g}{m}}, \quad \epsilon_{Tg} = \frac{14\sqrt{\pi}\epsilon_g}{9\text{Le}}, \quad \text{Le} \approx 1.4,$$

$$\epsilon_{vg} = \frac{14\sqrt{\pi}\epsilon_g}{9\text{Le}} \frac{\kappa_v}{\kappa_g}, \quad \epsilon_{Dg} = \frac{4\sqrt{\pi}\epsilon_g}{3\text{Sc}}, \quad \text{Sc} = 0.5, \quad \alpha_T, \alpha_v, \beta$$

are the accommodation and condensation coefficients. The expressions (2.4) encompass the whole range of possible changes in the Knudsen number and remain finite in appropriate limit cases. In fact, for a continuous medium when  $\epsilon_{Tg} \ll 1$ ,  $\epsilon_{vg} \ll 1$  for arbitrary values of the accommodation ( $0 \leq \alpha_T \leq 1$ ,  $0 \leq \alpha_v \leq 1$ ), we have  $\text{Nu} \rightarrow 2\alpha_T/(\alpha_T + \epsilon_{Tg}R^{-1})$ ,  $\text{Nu}_v \rightarrow 2\alpha_v/(\alpha_v + \epsilon_{vg}R^{-1})$ , and in the free molecule limit ( $\epsilon_g \rightarrow \infty$ ).

$$\text{Nu} \rightarrow \left( \frac{9\text{Le}}{7\sqrt{\pi}} \right) \frac{\alpha_T}{\epsilon_g}, \quad \text{Nu}_v \rightarrow \left( \frac{9\text{Le}}{7\sqrt{\pi}} \right) \frac{\alpha_v}{\epsilon_g}.$$

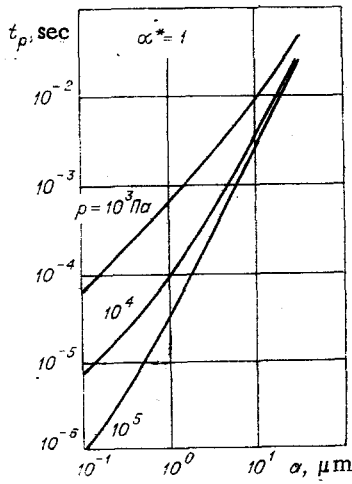


Fig. 3

For  $Re \rightarrow 0$  the problem of evaporation reduces to solving the system of two equations (2.1), (2.2). Linearizing the right side of (2.1) and using the relationship (2.4), we estimate the particle thermal equilibrium build-up time with the surrounding medium. Taking into account that the quantity is

$$q\mu_p/RT_g \gg 1, \quad (2.5)$$

it can be shown<sup>†</sup> that the particle size varies insignificantly in the time  $\tau_p$

$$dR/d\Theta_p \ll 1. \quad (2.6)$$

This permits the use of the known quasistationary approximation [6, 7, 9] according to which the particle surface temperature  $\Theta_p$  equals the equilibrium evaporation temperature  $\Theta^*$ , for the determination of the particle evaporation time. Therefore, to determine the desired  $\tau_p$  it is sufficient to find the solution of (2.2), which has the form  $\Theta_p = \Theta^* \approx \text{const}$ ,  $\alpha \ll 1$ ,

$$Le\alpha^*\tau = \frac{1-R^2}{2} + \varepsilon_g^2 \ln \frac{1+\varepsilon_g}{R+\varepsilon_g} + \left( \frac{\varepsilon_{Dg}}{\beta} - \varepsilon_g \right) (1-R)$$

because of the relationships (2.4) for  $\tau_p$ . Hence we obtain the expression

$$\tau_p = \frac{\frac{1}{2} + \varepsilon_g^2 \ln \frac{1+\varepsilon_g}{\varepsilon_g} + \frac{\varepsilon_{Dg}}{\beta} - \varepsilon_g}{Le\alpha^*} \approx \frac{2.8 \cdot 10^{-5}}{\alpha^*} \left( a^2 + 0.23 \frac{a}{\beta p_g} \right) \quad (2.7)$$

for the quantity  $p_g$ , where the magnitude of the gas phase pressure is referred to the value 101 kPa,  $\alpha = 10^6 R$ .

The dependence (2.7) is shown in Fig. 3 for the case  $\alpha^* = 1$ ,  $\beta \sim 1$ .

In contrast to the results known for the evaporation time [6, 7, 9], the expression (2.7) takes account of the influence of nonequilibrium excited vibrational degrees of freedom on the quantity  $\alpha^*$  and is valid for any  $\varepsilon_g$ . To obtain quantitative estimates of the influence of vibrational nonequilibrium on the quantity  $\alpha^*$  we analyze the solution of the heat balance equation in the following characteristic cases

$$\begin{aligned} \alpha_v &= 0, \quad \varepsilon_{Dg}/\varepsilon_{Tg} = \beta/\alpha_T, \\ 1 - \Theta^* &= Le\Theta^*\alpha^*(\Theta^*); \end{aligned} \quad (2.8)$$

$$\begin{aligned} \alpha_v &= \alpha_T, \quad \varepsilon_{Dg}/\varepsilon_{vg} = \beta/\alpha_v, \\ \Theta_{vg} &\simeq 10, \quad 3/2 - \Theta^* = Le\Theta\alpha^*(\Theta^*). \end{aligned} \quad (2.9)$$

Because of the inequality (2.5), the solution of the transcendental heat balance equations (2.8) and (2.9) can be found by successive approximations. The appropriate values of the coefficient  $\alpha^*$  are shown in the table for  $T_g = 300^\circ\text{K}$ , as calculated by using the data presented in [4].

<sup>†</sup>In particular, the quantity is  $q\mu_p/RT_g \geq 10$  for the flow modes under consideration for the two-phase  $\text{CO}_2\text{-N}_2$  mixture.

As follows from the table, taking account of the heat flux from the nonequilibrium vibrational degrees of freedom can significantly raise the value of the equilibrium aerosol vapor concentration  $\alpha^*$ . It should be noted that taking accurate account of this factor is difficult at this time since experimental measurements of the accommodation coefficient  $\alpha_v$  have been performed on a very small number of materials [10]. Nevertheless, the structure of (2.4) permits the assertion that in the solution for a continuous medium ( $\epsilon_g \ll 1$ ), even for low values of the accommodation coefficient  $\alpha_v \geq \epsilon_g$  and  $\Theta_{vg}\kappa_v/\kappa_g \sim 1$ , the heat flux from nonequilibrium vibrational degrees of freedom becomes commensurate with the magnitude of the heat flux from the translational and rotational degrees of freedom.

Now let us examine the mixing mode when  $Re \neq 0$  and the velocity of aerosol particle lagging behind the gas is subsonic.<sup>†</sup> The coefficients  $Nu$ ,  $Nu_v$ ,  $c_D$ ,  $Sh$  will hence also depend on the number  $Re$  [6, 7, 11]. It can be shown analogously to the preceding, that the temperature  $\Theta_p$  will equal the evaporation temperature  $\Theta^*$ . Hence, the problem again reduces to the system (2.2) and (2.3). The solution of these equations for the case of a continuous medium ( $\epsilon_g \ll 1$ ) was obtained in [7]. However, for  $\epsilon_g \geq 1$  the method used in [7] becomes unsuitable since the dependence of the coefficients  $Nu$  and  $c_D$  on the velocity  $V$  and the radius  $R$  is not a one-parameter dependence on the local Reynolds number any more. In this connection, to solve the problem taking account of the flow mode with finite values of the Knudsen number  $\epsilon_g$  in this paper, we use another approach. We use it on the simpler case when  $Nu_v = 0$ . On the basis of (2.2) and (2.3), we obtain for  $Nu_v = 0$

$$\frac{dR}{dV} = \frac{8H_1}{3Pr} \frac{Nu(Re, \epsilon_g, R)}{c'_D(Re, \epsilon_g, R)} \frac{R}{V}. \quad (2.10)$$

Here the dependence on the Knudsen number is taken into account in the coefficients  $Nu$ ,  $c'_D$  by the "inverse drag" rule [9, 11]:

$$Nu = \frac{Nu^c}{1 + 4.1\epsilon_g R^{-1} Nu^c}, \quad c'_D = \frac{24c_D^c}{1 + 2.5\epsilon_g R^{-1} c_D^c},$$

and the quantities  $Nu^c$ ,  $c_D^c$  are defined by the empirical dependences [6, 7, 11]

$$\begin{aligned} Nu^c &= H_1 [2 + 0.6Pr^{0.3} (ReVR)^{0.5}], \\ H_1 &= \ln(1 + B)/B, \\ B &= \frac{1 - \Theta_p}{Q}, \quad c_D^c = 1 + 0.48 (ReVR)^{0.5} + 0.04 (ReVR). \end{aligned}$$

The ratio of the coefficients  $Nu/c_D$  is a sufficiently conservative quantity in a broad range of numbers  $Re$  and  $\epsilon_g$  (for  $Re < \infty$ ,  $\epsilon_g \leq 10$ ) and  $\zeta = Nu/c_D \approx \text{const}$ . Hence, the approximate solution of (2.10) is written as

$$R = V^\omega. \quad (2.11)$$

Here  $\omega = [8/3][\ln(1 + B)/B]\zeta$ , for  $Pr \approx 0.8$  and  $\zeta = 0.1$ ,  $\omega \leq 0.3$ .

The expression (2.11) shows that the need for a joint solution of (2.2) and (2.3) is determined by the magnitude of the parameter  $B$ . For small values of  $B$ , when  $\omega \leq 0.1$ , the particle evaporation and deceleration processes proceed independently, where the deceleration process terminates earlier. Hence, for  $B \ll 1$  the particle evaporation time will agree with (2.7) obtained for the gas at rest. For high values of  $B$ , that can be due to the heat flux from the nonequilibrium degrees of freedom, in particular, the problems of particle evaporation and deceleration are generally connected. It is hence necessary to solve (2.2) with the dependences  $\tau_p$  and (2.11) taken into account to determine the particle evaporation time  $Nu(Re, \epsilon_g, R)$ . We then obtain

$$\tau_p = \frac{2Q}{1 - \Theta^*} \left[ a\epsilon_g + \frac{1}{4} \Phi(\delta, b\sqrt{Re}) \right]. \quad (2.12)$$

Here the parameter  $\delta = (1 + \omega)/4\omega$  does not exceed one since  $\omega < 1/3$ , where

$$\Phi(\delta, b\sqrt{Re}) = \begin{cases} F\left(1; \frac{1}{\tau}; \frac{1+\delta}{\delta}; -\frac{b\sqrt{Re}}{2}\right), & \delta < 1, \\ \frac{2}{b\sqrt{Re}} \ln\left(1 + \frac{b}{2}\sqrt{Re}\right), & \delta = 1. \end{cases}$$

<sup>†</sup>For  $w_{pt} \ll 1$  (see Sec. 1), the velocity of initial particle lag is supersonic. Analysis of this mode requires a separate examination.

TABLE 1

$P_g \cdot 101,$ kPa	$\alpha_v = 0$	$\alpha_v \approx \varepsilon_g$
	$\alpha^*$	$\alpha^*$
1	0,21	0,25
0,1	0,23	0,45
0,01	0,25	0,5

F is the hypergeometric function,  $\alpha = 4.1$ , and  $b = 0.6Pr^{0.33}$ .

For  $\omega \rightarrow 0$  values of the quantity (2.12) tend to the appropriate values for a gas at rest, and for

$$(b\sqrt{\text{Re}})^{1/\delta} \gg 1 \quad (2.13)$$

the values of  $\tau_p$  are determined completely by the particle deceleration time, where for  $\delta < 1$  and  $(b\sqrt{\text{Re}})^{1/\delta} \rightarrow \infty$

$$\Phi \rightarrow \left( \frac{2}{b\sqrt{\text{Re}}} \right)^{1/\delta} \frac{\pi}{\delta} \text{cosec} \frac{\pi}{\delta}.$$

It can be concluded on the basis of the inequality (2.13) that for moderate values of the number  $\text{Re} (\text{Re} \sim 10)$ , the time  $\tau_p$  can be estimated by the gas at rest.

Furthermore, let us examine the question of the depth of aerosol particle penetration into the supersonic stream. As the particles increase by a uniform gas stream ( $v_g = \text{const}$ ), the relative motion trajectory is, as is shown in [6], a straight line at a constant angle  $\mu$  to the gas flow direction  $v_g$ . Hence, to determine the depth of aerosol particle penetration  $L_p$  into the gas stream it is sufficient to find the projection of the total path  $S_p$  of particle deceleration in a direction orthogonal to the velocity direction  $v_g$

$$L_p = S_p \sin \mu,$$

where

$$\frac{S_p}{V_0 \tau_p} = \int_1^0 \frac{v dv}{dv/dt}; \quad \sin \mu = \frac{v_p \sin \beta}{V_0};$$

$\beta$  is the angle between the gas  $v_g$  and the aerosol particle  $v_p$  velocity vectors at the initial time  $\tau = 0$ ,  $V_0 = |v_g - v_p|_{\tau=0}$ .

We use the correlation dependence for the coefficient  $c_p^1$  (see (2.10)) and the relationship (2.11) to determine the quantity  $S_p$ . We then obtain

$$S_p \approx \frac{2}{9} a^2 10^3 \left( 1 + \frac{3.6 \cdot 10^{-2}}{p_g^a} \right). \quad (2.14)$$

The dependence (2.14) is presented in Fig. 4 for the case  $\alpha^* = 1$ ,  $\beta = 1$ ,  $V_0 = |v_g - v_p| = 100$  m/sec. Similar results, characterizing the deceleration path  $S_p$  and the lifetime of particles of different diameters in a stream, are shown in Fig. 5 as a function of the pressure ( $\alpha^* = 1$ ,  $V_0 = 500$  m/sec). The results obtained indicate that particles with the radii  $r_p > 1 \mu\text{m}$  satisfy the condition of sufficiently deep penetration in the  $p = (0.1-0.01) \cdot 10^5$  Pa pressure range.

An important question is the organization of fast and homogeneous mixing of the  $\text{CO}_2$  vapor being formed and the  $\text{N}_2$  molecules. "Instantaneous" mixing assumes that this should be realized in the time scale of the vibrational relaxation  $\tau_m$ . Mixing occurs in the mentioned range of numbers  $\text{Re}$  because of laminar diffusion on an interparticle spacing scale. The process of blowing vapor away from the particle surface by a coflow during particle acceleration does not change this characteristic scale. The condition  $\tau_D \leq \tau_m$  should evidently be satisfied for the diffusion time.

Despite the sufficient "rigidity" of this conditions, it does not in itself solve completely the question of uniform mixing.

Since evaporation occurs nonuniformly with the diminution of the particle radius, the vapor distribution over the section can have a certain profile at the end of the mixing zone. The solution of this problem requires a separate examination. The authors are grateful to A. M. Prokhorov for turning their attention to the questions considered in the paper, and for discussing these questions.

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#### NUMERICAL INVESTIGATIONS OF THE INFLUENCE OF FRICTION ON THE MAGNITUDE OF LOSSES IN A NOZZLE CASCADE OF A GASDYNAMIC CO<sub>2</sub>-N<sub>2</sub>-He LASER

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Sets of short nozzles (nozzle cascades) with small critical sections [1] are used to obtain strongly nonequilibrium gas flows in gasdynamic CO<sub>2</sub> lasers. The gas viscosity plays a relatively large part in such apparatus. The friction-caused losses in gasdynamic lasers (GDL) were determined by numerical methods in [2, 3]. It was assumed in both papers that the interactions between different kinds of vibrations in the gas molecules and the nozzle surface are simultaneously subject to the same regularities. However, multiatomic molecules have a complex spatial structure as a rule, which should generally result in different relaxation rates for their vibrational degrees of freedom on the boundary of contiguity of two phases. The question of the possible practical utilization of these phenomena was discussed in [4]. Later, in [5] a mathematical model of CO<sub>2</sub> molecule relaxation on the surface of aqueous aerosol particles [4] was proposed for utilization in the analysis of physical processes in GDL.

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